JMEPEG (2008) 17:472–481
DOI: 10.1007/s11665-007-9172-6

1059-9495/$19.00

Linear and Non-linear Modeling of Cement-bonded Moulding Sand System Using Conventional Statistical Regression Analysis

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(Submitted January 18, 2007; in revised form August 2, 2007)

A cement-bonded moulding sand system takes a fairly long time to attain the required strength. Hence, the moulds prepared with cement as a bonding material will have to wait a long time for the metal to be poured. In this work, an accelerator was used to accelerate the process of developing the bonding strength. Regression analysis was carried out on the experimental data collected as per statistical design of experiments (DOE) to establish input-output relationships of the process. The experiments were conducted to measure compression strength and hardness (output parameters) by varying the input variables, namely amount of cement, amount of accelerator, water in the form of cement-to-water ratio, and testing time. A two-level full-factorial design was used for linear regression model, whereas a three-level central composite design (CCD) had been utilized to develop non-linear regression model. Surface plots and main effects plots were used to study the effects of amount of cement, amount of accelerator, water and testing time on compression strength, and mould hardness. It was observed from both the linear as well as non-linear models that amount of cement, accelerator, and testing time have some positive contributions, whereas cement-to-water ratio has negative contribution to both the above responses. Compression strength was found to have linear relationship with the amount of cement and accelerator, and non-linear relationship with the remaining process parameters. Mould hardness was seen to vary linearly with testing time and non-linearly with the other parameters. Analysis of variance (ANOVA) was performed to test statistical adequacy of the models. Twenty random test cases were considered to test and compare their performances. Non-linear regression models were found to perform better than the linear models for both the responses. An attempt was also made to express compression strength of the moulding sand system as a function of mould hardness.

Keywords regression analysis, DOE, CCD, full-factorial design, ANOVA

1. Introduction

Cement-bonded moulds can be made with considerable precision, resulting in more accurate castings (Ref 1). Another advantage of using cement as a bonding material is that no gas is generated, when the molten metal comes in contact with the cement. Cement-bonded moulds are often proved to be more economical for large size castings, as the saving results in labor and post-cleaning operations (Ref 2). The main drawback of a cement-bonded mould is its slow hardening rate. It requires generally 6-8 h to attain minimum compression strength, at which pattern can be withdrawn from the mould and around 72 h to attain the maximum strength, at which the molten metal can be poured (Ref 3, 4).

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Cement-bonded moulding sand is a mixture of silica sand, Portland cement, and water. The sand develops the hardness and strength due to the setting action of Portland cement. The major constituents of cement are tricalcium silicate ($3CaO \cdot SiO_2$), dicalcium silicate ($2CaO \cdot SiO_2$), tricalcium aluminate ($3CaO \cdot Al_2O_3$), and tetracalcium aluminoferrite ($4CaO \cdot Al_2O_3 \cdot Fe_2O_3$). In the presence of water, silicates, and aluminates form the products of hydration over a period of time and produce a hard mass-hydrated cement paste. The rate of hydration decreases continuously, so that even after a long time, there remains an appreciable amount of unhydrated cement. This may be due to the formation of protective layer of hydrated cement, which prevents further chemical reaction (Ref 3, 5). The rate of hydration of cement particles can be increased considerably by using an accelerator, so that the strength may be obtained early in the cement-bonded moulding sand mixture. Accelerator is a chemical product and complies with ASTM type C admixtures. Various types of accelerators are commercially available for use in concrete mixtures. Calcium chloride, sodium chloride, calcium nitrite, calcium nitrate, calcium aluminate, calcium formate, and sodium formate are some of the accelerators used in concrete mixtures (Ref 5). Chlorine-based accelerators may react with the iron content of the ferrous castings. Thus, those were not tried in the present study. Uchikawa and Uchida (Ref 6) used the accelerators containing calcium hydroxide and calcium aluminate to develop the early strength in cement-bonded moulds. Based on the above literature and depending on the availability, calcium formate ($Ca[OOOCH]_2$) was chosen as an accelerator for the cement-bonded moulding sand system.

Statistical design of experiment (DOE) refers to the process of planning the experiment, so that an appropriate set of data can be collected, and then analyzed using the regression analysis for drawing inferences on the input-output relationships of a system (Ref 7). The DOE combined with response surface methodology is a powerful tool to develop the relationships. Response surface methodology is an empirical modeling approach using polynomials as local approximations to the true input-output relationships. Full-factorial design of experiments with the parameters set at their respective two levels, can be used to develop linear relationships between the input-output parameters. The number of experiments required is less and the analysis provides a complete information on the main and interaction effects of input parameters on the response. The main problem with this linear model lies in the fact that the curvature effect (if any) in the response function will not be recognized. On the other hand, central composite design (CCD) can be used to develop the non-linear models (Ref 8). As the interdependency of the output responses cannot be considered in the conventional statistical regression models, a separate model can be developed for each response and the best of the statistical models can be used to predict the response. The statistical models developed through DOE and response surface methodology are accurate and can be used to predict the responses. The CCD is called rotatable, if the accuracy in prediction of a response is the same on a sphere around the center of the design. Rotatable designs require the input parameters to be set at their five levels, whereas non-rotatable designs require the parameters to be set at three levels only. The choice of a rotatable or non-rotatable design depends on the geometric nature of the practical constraints on the design region. The levels of input variables are set independent to each other, hence, the chosen design is a cuboidal non-rotatable central composite design. Unless some practical considerations dictate the choice of a rotatable design, it should not be strictly followed (Ref 8, 9).

The amount of literature available on cement-bonded moulding sand system is less compared to that available on other moulding sand systems, although it has a practical value as mentioned above. Sleicher and Providence (Ref 3) explained the advantages and drawbacks of using Portland cement as a binder in making sand moulds. Some work had been carried out in Japan during 1970s to explore the possibilities of using cement as the bonding material. Uchikawa and Uchida (Ref 6) used the classical approach to study the effects of variables on cement-bonded sand moulding system. They used quick setting cement (jet cement) with accelerator to overcome the drawbacks of ordinary Portland cement. In their work, the effects of amount of jet cement, testing time, presence of an accelerator and amount of water on compression strength were studied by varying one variable at a time and keeping the others fixed. To the best of the present authors' knowledge, no work has been reported on regression analysis of cement-bonded moulding sand system.

The present work is aimed at using the conventional statistical tools like DOE and Response Surface Methodology (RSM) to develop the input-output relationships in cement-bonded moulding sand systems. An attempt was also made to develop the strength of the cement-bonded moulding sand system early using an accelerator (calcium formate). The complex relationships of the input variables on mould properties were analyzed with the help of main effects plots and surface plots.

In this work, the following things were attempted:

(i) An accelerator was used to ensure quick setting of the developed cement-bonded moulding sand system.
(ii) Experiments were conducted as per the requirement of full-factorial design. Linear models were developed for the responses, compression strength and mould hardness. Adequacy of the developed models was checked by statistical analysis and their performances were tested on 20 random test cases.
(iii) Non-rotatable CCD was used to develop non-linear models for both the responses. Statistical analysis was performed and adequacy of the models was checked.
(iv) The performances of linear and non-linear models were compared with the help of some randomly developed test cases (refer to Appendix C) and the best model having the minimum % deviation in prediction was identified for each response.
(v) Relationship between the responses, compression strength and mould hardness, was developed.

2. Methodology

The aim of the present work is to establish the input-output relationships of a cement-bonded sand mould system as shown in Fig. 1.

2.1 Selecting Process Parameters and Their Levels

Washed and dried high silica sand was used in the experiments. The sieve analysis test showed that 80% of sand was retained on three consecutive sieves with a single peak distribution. The fineness number of the sand was found to be equal to 52 (AFS NO. 52).

The suitable ranges of different process parameters used in the experiments are discussed below.

2.1.1 Portland Cement. Ordinary Portland cement (43 grade ACC) was used as binder. The levels of Portland cement were kept fixed at 8%, 10%, and 12% of silica sand, after consulting the literature (Ref 2, 3, 5).

2.1.2 Accelerator. A chlorine-free accelerator, namely Conplast NC containing calcium formate ($Ca[OOOCH]_2$) as the main ingredient was used in the experiments. This accelerator complies with BS 5075 Part 1 and ASTM C494 Type C. The levels of accelerator were set at 2%, 3%, and 4% of cement based on the literature (Ref 5, 6) and manufacturer's catalogue. The addition of accelerator amounting more than 4% of cement will not help for further improvement in developing the early strength.

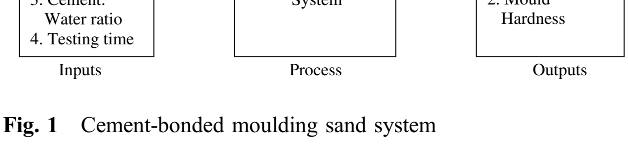

Fig. 1 Cement-bonded moulding sand system

2.1.3 Cement-to-Water Ratio. The water required to activate the cement is dependent on the amount of cement. Hence, the variable water is considered in the form of cement-to-water ratio. The levels of this variable were selected based on the literature (Ref 2, 3, 5) as 1.5, 2.0, and 2.5.

2.1.4 Testing Time. It is the time elapsed between sample preparation and its testing. A number of trial runs were made and the testing time intervals were kept fixed to 2, 5, and 8 h.

The input parameters and their levels used in the experiments are shown in Table 1.

2.2 Conducting Experiments

Experiments were conducted as per the design matrices of full-factorial design and central composite design, as shown in Appendice A and B, respectively. The test specimens were prepared as per the standard procedure. Accelerator was added to the silica sand and cement mixture along with water. The compression strength and mould hardness were measured using a universal strength testing machine and mould hardness tester, respectively. Four replicates were considered for each combination of the input variables and test cases.

2.3 Developing the Statistical Models, Statistical Analysis, and Testing of the Models

Linear and non-linear regression analysis was carried out using the data collected as per 2-level full-factorial design and 3-level central composite design of experiments, respectively, for the responses, compression strength and hardness. Significant tests were carried out to examine the effect of the parameters and their interaction terms. Analysis of variance (ANOVA) was conducted for each of the responses to check the adequacy of the models. The detailed analysis of the effects of parameters and their interactions on the responses was also done through the main effect plots and surface plots. MINITAB software was used for this purpose. Further, the performances of the models were tested with the help of 20 test cases generated at random.

3. Results and Discussions

The experimental data collected were used to develop linear regression model based on full-factorial design and non-linear regression model based on CCD using MINITAB software.

3.1 Statistical Models and Analysis

The statistical analysis of the developed regression models was performed through the significance and ANOVA tests. The input-output relations were studied with the help of main effect plots and surface plots for the responses, compression strength and mould hardness.

3.1.1 Response-Compression Strength. The linear model based on full-factorial design and non-linear regression model based on CCD were developed for the response-compression strength, as explained below.

Linear Model Based on 2-Level Full-Factorial Design. The linear input-output relationship for the response-compression strength is shown in the following equation:

$$\begin{aligned} CS_{fact} = {} & 66.59 + 28.11X_1 + 13.30X_2 - 24.81X_3 + 36.92X_4 \\ & + 5.51X_1X_2 - 4.13X_1X_3 + 15.67X_1X_4 - 2.04X_2X_3 \\ & + 6.39X_2X_4 - 11.60X_3X_4 - 2.73X_1X_2X_3 \\ & + 2.47X_1X_2X_4 - 0.48X_1X_3X_4 + 1.78X_2X_3X_4 \\ & - 0.81X_1X_2X_3X_4 \qquad \text{(Eq 1)} \end{aligned}$$

Significance test was carried out to study the effects, contributions and significance of the input parameters and their interaction terms on the response-compression strength. The significance test results are shown in Table 2.

The different terms used in Table 2 have been explained as follows: The term 'Coef' indicates the coefficients used in Eq 1 for representing the relationship between the said response parameter and the factors. The term 'SE Coeff' represents the standard error for the estimated coefficient, which measures the precision of the estimate. The smaller the standard error, the more precise will be the coefficient. The *T*-values are calculated as the ratio of corresponding value under coefficient and standard error. The *T*-value of the independent variable can be used to test, whether the predictor significantly affects the response. The *p*-value is the minimum value for a pre-set level

Table 2 Results of significance test on the linear model-effects, coefficients, *T*-statistics and *p* values for the response-compression strength

Term	Effect	Coef	SE Coef	*T*	*p*
Constant		66.59	0.4163	159.97	0
X_1	56.21	28.11	0.4163	67.52	0
X_2	26.61	13.3	0.4163	31.96	0
X_3	−49.62	−24.81	0.4163	−59.6	0
X_4	73.84	36.92	0.4163	88.69	0
X_1X_2	11.01	5.51	0.4163	13.22	0
X_1X_3	−8.25	−4.13	0.4163	−9.91	0
X_1X_4	31.35	15.67	0.4163	37.66	0
X_2X_3	−4.07	−2.04	0.4163	−4.89	0
X_2X_4	12.78	6.39	0.4163	15.35	0
X_3X_4	−23.21	−11.6	0.4163	−27.87	0
$X_1X_2X_3$	−5.45	−2.73	0.4163	−6.55	0
$X_1X_2X_4$	4.93	2.47	0.4163	5.93	0
$X_1X_3X_4$	−0.97	−0.48	0.4163	−1.16	0.25
$X_2X_3X_4$	3.56	1.78	0.4163	4.27	0
$X_1X_2X_3X_4$	−1.62	−0.81	0.4163	−1.94	0.058

Table 1 Process parameters and their levels

Input parameters	Notation		Low level (−1)	Middle level (0)	High level (+1)
	Un-coded	Coded			
Portland cement	*A*	X_1	8% of silica sand	10% of silica sand	12% of silica sand
Accelerator	*B*	X_2	2% of cement	3% of cement	4% of cement
Cement:water ratio	*C*	X_3	1.5	2.0	2.5
Testing time (h)	*D*	X_4	2	5	8

of significance, at which the hypothesis of equal means for a given factor can be rejected. Considering 95% level of confidence, the significance of different factors and their interaction terms were tested.

As the p values (refer to Table 2) of the terms $X_1X_3X_4$ and $X_1X_2X_3X_4$ were found to be more than 0.05, those were considered to have no significant contributions on the response-compression strength at 95% confidence level, whereas all other main and interaction terms shown in Table 2 were found to have significant contributions. From the above table, it was seen that the parameter-testing time (X_4) has the maximum positive contribution, whereas the amount of cement-to-water ratio (X_3) was found to have maximum negative contribution. Among the interaction terms, X_1X_4, i.e., amount of Portland cement and testing time interaction had the maximum positive contribution, whereas cement-to-water and testing time (X_3X_4) had the maximum negative contribution on the said response.

The contributions of the input variables on the response-compression strength are shown in Fig. 2. Amount of cement, amount of accelerator and testing time were seen to have positive contributions, i.e., increase in the level of these variables will increase the response-compression strength, whereas cement-to-water ratio had negative contribution on the said response. The input parameters were coded using the following relationships:

$$X_1 = \frac{A-10}{2}, \quad X_2 = \frac{B-3}{1}, \quad X_3 = \frac{C-2}{0.5}, \quad X_4 = \frac{D-5}{3}$$

where X_1, X_2, X_3, and X_4 represent the input parameters, such as % Portland cement A, % accelerator B, cement-to-water ratio C and testing time D, respectively, in the coded form.

The response equation can be written in the un-coded form as follows:

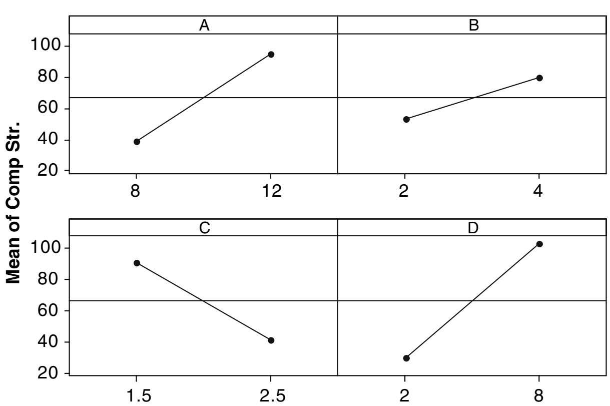

Fig. 2 Main effects plot for the response-compression strength

$$\begin{aligned} CS_{fact} = & -13.6459 - 2.73636A - 11.8935B + 10.8592C \\ & + 27.6365D + 3.45456AB + 0.81875AC \\ & + 0.086184AD + 3.79212BC - 9.73884BD \\ & - 17.7540CD - 1.37895ABC + 0.949825ABD \\ & + 0.646383ACD + 3.87830BCD - 0.269326ABCD \end{aligned} \quad \text{(Eq 2)}$$

Analysis of variance (ANOVA) was performed to test the significance of the factors for the response-compression strength. The results of ANOVA are shown in Table 3. The different terms used in this table have been explained below. The term 'DF' represents the degrees of freedom. Degree of freedom indicates the number of terms that will contribute to the error in prediction. The term 'Seq SS' indicates the sum of squares for each term, which measures the variability in the data contributed by that term. The adjusted sum of squares (i.e., Adj SS) is the sum of squares obtained after removing insignificant terms from the model. The sum of squares is divided by the degrees of freedom to determine the mean square (MS). The adjusted mean square (i.e., Adj MS) is the mean square obtained after removing the insignificant terms from the response equation. The 'F' value for regression is used to test the hypothesis, which is calculated as the ratio of adjusted mean square value to the residual error.

From the ANOVA table, it is to be noted that four-factor interaction term was found to be insignificant for this response. The coefficient of correlation for this model was found to be equal to 0.998. The results of ANOVA and the coefficient of correlation indicate that the developed linear regression model based on full-factorial design is statistically adequate.

Non-linear Model Based on Central Composite Design (CCD). The experimental data collected as per the central composite design was utilized to develop the non-linear regression model for the response-compression strength. The response equation with the parameters expressed in the coded form was found to be as follows:

$$\begin{aligned} CS_{ccd} = & \ 61.8272 + 27.5024X_1 + 12.7936X_2 - 23.9210X_3 \\ & + 37.2125X_4 + 0.8555X_1^2 - 3.2814X_2^2 - 5.6946X_3^2 \\ & + 12.5765X_4^2 + 5.5050X_1X_2 - 4.1261X_1X_3 \\ & + 15.6748X_1X_4 - 2.0361X_2X_3 + 6.3884X_2X_4 \\ & - 11.6026X_3X_4 \end{aligned} \quad \text{(Eq 3)}$$

The significance test of this model was conducted for the response-compression strength. The p values of the square terms of the input variables, such as % cement and % accelerator (i.e., X_1^2 and X_2^2, respectively) were found to be more than 0.05. It indicates that these factors are insignificant and

Table 3 Results of ANOVA for the response-compression strength

Source	DF	Seq SS	Adj SS	Adj MS	F	p
Main effects	4	188518	188518	47129.6	4249.87	0
2-Way interactions	6	30247	30247	5041.1	454.58	0
3-Way interactions	4	1082	1082	270.6	24.4	0
4-Way interactions	1	42	42	41.8	3.77	0.058
Residual error	48	532	532	11.1		
Pure error	48	532	532	11.1		
Total	63	220422				

their relationship with the response-compression strength is linear. All other terms in the response equation were found to be significant, as their p values were seen to be less than 0.05.

The surface plots help to visualize the relationship of input variables on the response. The surface plots for the response-compression strength are shown in Fig. 3. The following observations were made from the above plots for this response:

(i) Figure 3(a) shows that the response-compression strength will increase with the increase of the amount of Portland cement and accelerator. The response surface was found to be almost a flat one, which indicates strong linear relationship of these input parameters with the response-compression strength. The increase in the amount of cement would provide with more bonding action, whereas increase in the amount of accelerator would cause rapid hydration of the cement resulting in high compression strength.

(ii) An increase in the amount of cement would increase the compression strength linearly, whereas an increase in cement-to-water ratio (i.e., decrease in water quantity) will reduce the response-compression strength (refer to Fig. 3(b)). The less the amount of water, the less would be the hydrated cement particles resulting in a lower compression strength.

(iii) Figure 3(c) shows that there would be a rapid increase in compression strength with the increase of amount of cement and testing time. This increase in strength could be due to more hydration of the cement over a long period of time.

(iv) An increase in the amount of accelerator would increase the compression strength, whereas the increase in cement-to-water ratio would reduce the compression strength (refer to Fig. 3(d)). The resulting response surface shows a slight curvature due to a non-linear relationship of cement-to-water ratio with the response-compression strength.

(v) Compression strength would increase rapidly with the increase of the amount of accelerator and testing time as shown in Fig. 3(e). The figure shows that the contribution of testing time toward this response would be more compared to that of the accelerator.

(vi) The increase in cement-to-water ratio would reduce the compression strength, whereas the testing time would

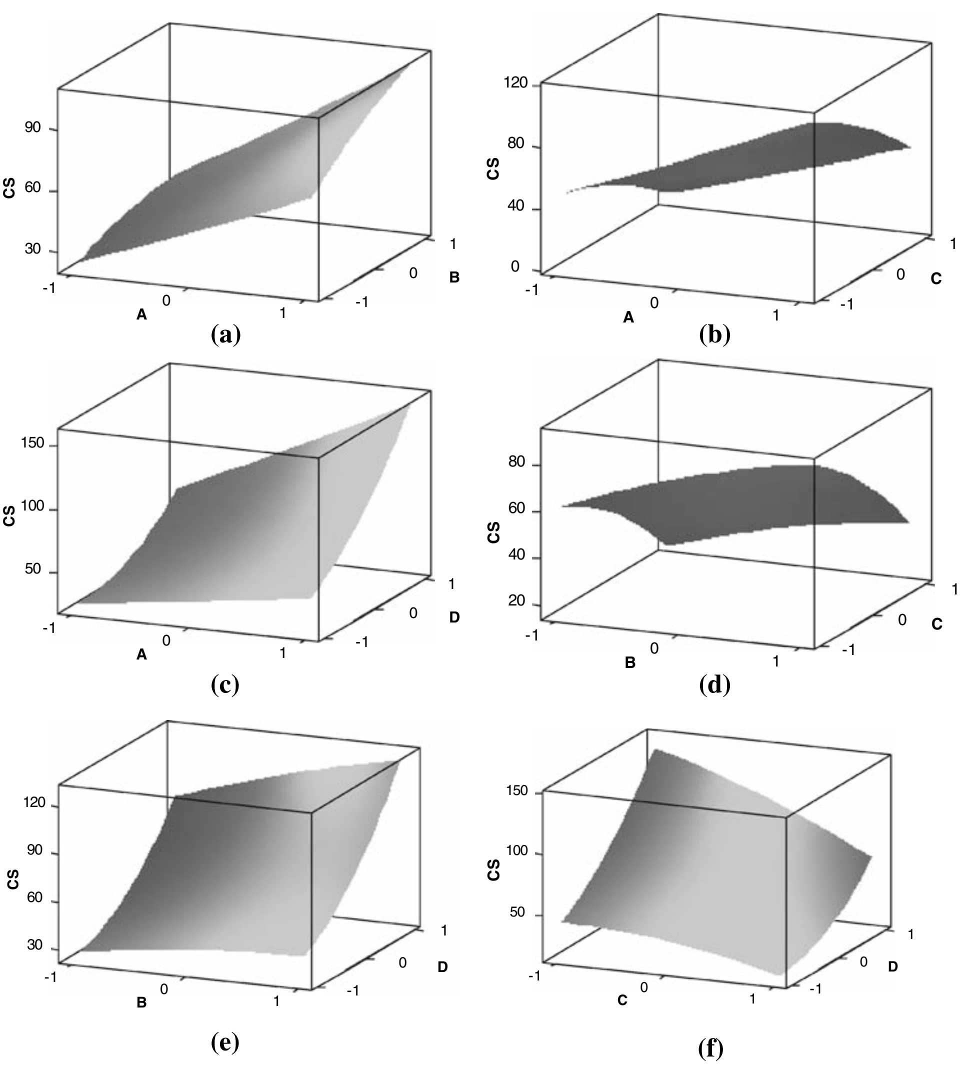

Fig. 3 Surface plots of compression strength with (a) % cement and accelerator, (b) % cement and cement-to-water ratio, (c) % cement and testing time, (d) accelerator and cement-to-water ratio, (e) accelerator and testing time, and (f) cement-to-water ratio and testing time

increase the response value (refer to Fig. 3(f)). The resulting surface plot is non-linear in nature.

The un-coded form of this response was found to be as follows:

$$
\begin{aligned}
CS_{ccd} = & -83.8913 - 3.59381A - 2.45386B + 135.424C \\
& - 18.6127D + 0.213865A^2 - 3.28139B^2 - 22.7782C^2 \\
& + 1.39739D^2 + 2.75252AB - 4.12608AC \\
& + 2.61247AD - 4.07222BC + 2.12947BD \\
& - 7.73506CD
\end{aligned} \quad \text{(Eq 4)}
$$

Analysis of variance (ANOVA) was performed to test the significance of this response. All the linear, square and interaction terms were found to be significant, as their p values were seen to be less than 0.05. It is important to mention that the lack of fit becomes significant, if the insignificant terms are removed from the response equation. The coefficient of correlation was found to be equal to 0.992. The above discussion indicates that the non-linear regression model developed based on CCD is statistically adequate and hence could be used in predicting the response.

3.1.2 Response-Mould Hardness. The experimental data collected as per the 2-level full-factorial design and central composite design (CCD) were used to develop linear and non-linear regression models, respectively.

Linear Model Based on 2-Level Full-Factorial Design. It can be observed from the significance test carried out for the linear regression analysis (coded form) that the input parameter-testing time (X_4) had the maximum positive contribution, whereas cement-to-water ratio (X_3) had the maximum negative contribution on the response-mould hardness, among the main factors. The p values of the interaction terms X_1X_4 (i.e., % cement and testing time), $X_1X_2X_4$ (i.e., % cement, % accelerator and testing time) and $X_1X_3X_4$ (i.e., % cement, cement-to-water ratio, and testing time) were found to be more than 0.05, indicating their insignificant contributions toward the response-mould hardness.

The linear regression equation for mould hardness expressed in its un-coded form was found to be as follows:

$$
\begin{aligned}
H_{fact} = & 142.542 - 5.56250A - 12.0625B - 51.0833C \\
& + 6.16667D + 1.34896AB + 4.04167AC - 0.375AD \\
& + 7.79167BC - 2.0BD - 2.83333CD - 0.760417ABC \\
& + 0.153646ABD + 0.229167ACD + 1.16667BCD \\
& - 0.0885417ABCD
\end{aligned} \quad \text{(Eq 5)}
$$

The results of ANOVA show that all the main factors, 2-way interaction, 3-way interaction, and 4-way interaction terms of the linear regression model for this response were significant. The coefficient of correlation was found to be equal to 0.998 for the said response.

Non-linear Model Based on Central Composite Design (CCD). The significance test for this analysis indicates that all linear terms were significant, as their p values were found to be less than 0.05. Among the square terms, testing time, i.e., X_4 was found to be insignificant indicating its linear relationship with the mould hardness. The interaction of % cement and testing time (X_1X_4), % accelerator and testing time (X_2X_4) were also found to be insignificant.

The surface plots and main effect plots of mould hardness showed almost the similar nature as that of compression strength. This might be due to the fact that compression strength and mould hardness are closely related.

The un-coded form of non-linear regression equation for mould hardness developed based on CCD was found to be as follows:

$$
\begin{aligned}
H_{ccd} = & 13.5199 + 7.38194A + 11.6111B + 6.76042C \\
& + 0.113812D - 0.388889A^2 - 1.68056B^2 \\
& - 9.22222C^2 + 0.0493827D^2 - 0.289062AB \\
& + 1.57813AC + 0.0130208AD + 1.59375BC \\
& + 0.0989583BD + 0.302083CD
\end{aligned} \quad \text{(Eq 6)}
$$

All linear, square, and interaction terms of the non-linear model developed based on CCD for this response, were seen to be significant as indicated by the results of ANOVA test. The lack of fit would become significant with the removal of insignificant terms from the model.

The coefficient of correlation was found to be equal to 0.979. From the above discussions on statistical analysis, it may be concluded that the non-linear regression model developed based on CCD for this response, is statistically adequate and can be used for predicting the response.

3.2 Testing of the Model

The detailed statistical analysis was performed and statistical adequacy of the models was checked in the previous sub-sections. Further, the performances of the developed models were tested on twenty test cases generated at random by considering different combinations of the input variables (refer to Appendix C).

3.2.1 Response-Compression Strength. Figure 4(a) and (b) compare the actual and predicted values of the response-compression strength obtained by the linear and non-linear regression models, respectively. The performance of the non-linear regression model was found to be better than that of the linear regression model, as the best-fit line for the former was seen to be closer to the ideal line ($y = x$). The values of % deviation in prediction of compression strength for 20 test cases are shown in Fig. 4(c). The values of % deviation in prediction were found to lie in the ranges of −44.12% to + 10.66% for the linear regression model and −14.91% to + 12.06% for the CCD-based non-linear model. For this response, the non-linear model was found to perform better than linear model and this could be due to the fact that the input-output relationship is non-linear in nature.

The summary of the test results discussed above for this response is shown in Table 4.

3.2.2 Response-Mould Hardness. The best-fit line passing through the points, which were plotted for making comparisons of the experimental mould hardness values with the model-predicted values for the linear regression and non-linear regression models are shown in Fig. 5(a) and (b), respectively. The best-fit line obtained for the linear model showed a large deviation from the ideal line, whereas non-linear model yielded the best-fit line, which was found to be very close to the ideal line with the uniform distribution of points on both sides of the best-fit line. Figure 5(c) shows the % deviation in prediction of mould hardness, as obtained by the linear regression and non-linear regression models. The values

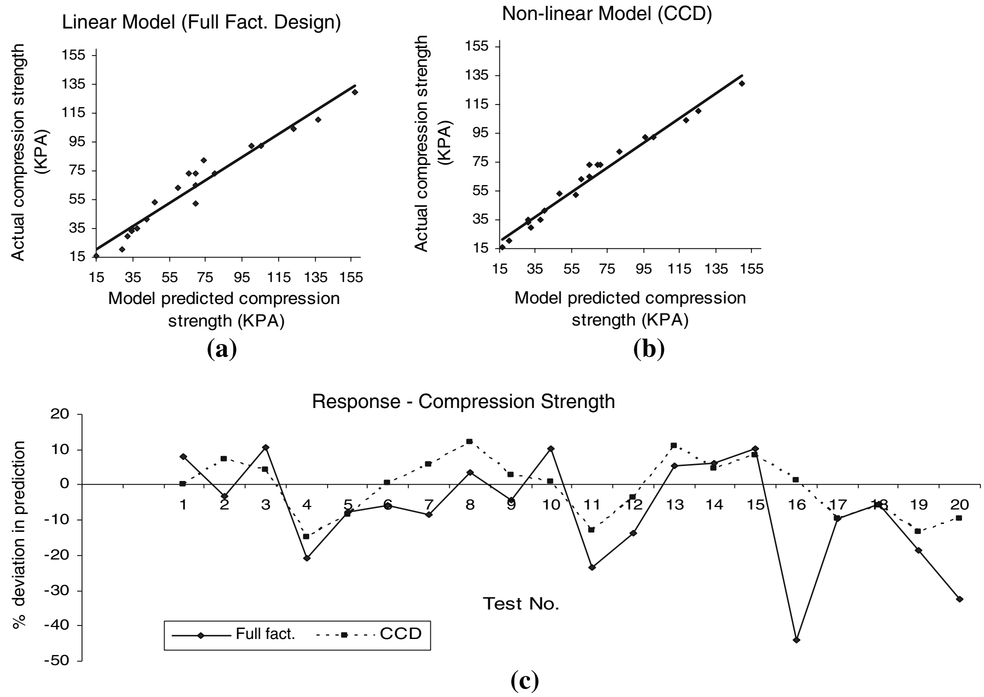

Fig. 4 Comparison of the model-predicted values of compression strength with the experimental results for 20 test cases: (a) actual compression strength vs. model-predicted compression strength values: linear model, (b) compression strength vs. model-predicted compression strength values: non-linear model, and (c) the values of % deviation in prediction of compression strength

Table 4 Summary of the results of test cases for the response-compression strength

		Compression strength using full-factorial design, kPa				Compression strength using central composite design, kPa			
Test No.	Measured CS, kPa	Full-Fact. design	Deviation	% Deviation	Abs % deviation	CCD	Deviation	% Deviation	Abs % deviation
1	16.20	14.91	1.29	7.98	7.98	16.17	0.03	0.17	0.17
2	33.61	34.65	−1.03	−3.08	3.08	31.16	2.45	7.30	7.30
3	73.77	65.91	7.86	10.66	10.66	70.53	3.24	4.39	4.39
4	130.14	157.06	−26.92	−20.69	20.69	149.54	−19.40	−14.91	14.91
5	92.73	99.91	−7.18	−7.74	7.74	100.69	−7.96	−8.58	8.58
6	65.33	69.23	−3.91	−5.98	5.98	65.05	0.28	0.43	0.43
7	73.60	79.81	−6.21	−8.43	8.43	69.43	4.18	5.67	5.67
8	35.51	34.22	1.29	3.64	3.64	31.22	4.28	12.06	12.06
9	41.02	42.84	−1.82	−4.43	4.43	39.90	1.13	2.75	2.75
10	82.05	73.76	8.28	10.10	10.10	81.21	0.84	1.02	1.02
11	111.01	137.15	−26.15	−23.55	23.55	125.40	-14.39	−12.97	12.97
12	92.56	105.33	−12.77	−13.80	13.80	95.82	−3.26	−3.52	3.52
13	73.08	69.23	3.85	5.27	5.27	65.05	8.03	10.99	10.99
14	63.43	59.63	3.80	5.99	5.99	60.52	2.91	4.59	4.59
15	52.92	47.44	5.48	10.36	10.36	48.54	4.38	8.28	8.28
16	20.17	29.07	−8.90	−44.12	44.12	19.94	0.23	1.13	1.13
17	29.30	32.15	−2.84	−9.71	9.71	32.09	−2.79	−9.52	9.52
18	35.51	37.45	−1.94	−5.48	5.48	37.52	−2.01	−5.67	5.67
19	104.28	123.75	−19.46	−18.66	18.66	118.19	−13.90	−13.33	13.33
20	52.57	69.67	−17.10	−32.52	32.52	57.60	−5.02	−9.55	9.55
Average of absolute % deviation in prediction				12.61					6.84

of % deviation in prediction were found to vary from −0.49% to +7.42%, in case of linear regression model, whereas the % deviation values were seen to lie within in the range of −2.60% to +3.22% in case of CCD-based non-linear model. The better performance of non-linear regression model compared to that of the linear regression model might have come due to the highly non-linear input-output relationships of the process.

Table 5 provides with the summary of the test results for the response-mould hardness.

3.3 Comparison of the Performance of Linear and Non-linear Models

Figure 6 compares the performance of the linear regression model with that of the non-linear regression model for

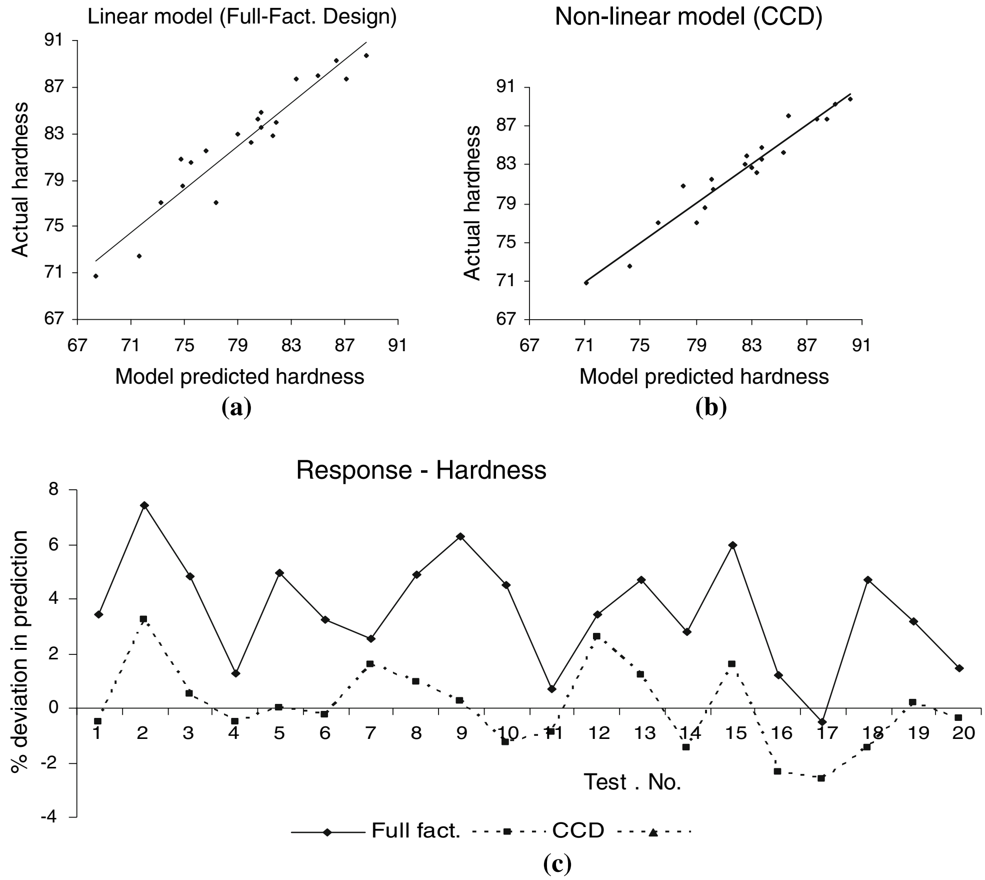

Fig. 5 Comparison of the model-predicted values of mould hardness with the experimental results for 20 test cases: (a) actual mould hardness vs. model-predicted mould hardness values: linear model, (b) mould hardness vs. model-predicted mould hardness values: non-linear model, and (c) the values of % deviation in prediction of mould hardness

predicting the responses, compression strength and mould hardness. The non-linear model was found to outperform the linear regression model for both the responses indicating most probable non-linear input-output relationships of the process.

3.4 Relationship Between Compression Strength and Hardness of the Mould

The following third-order non-linear relationship was obtained between the compression strength CS and mould hardness H:

$$\mathrm{CS} = -4068.88005 + 170.7116H - 2.40059H^2 + 0.01136H^3 \quad \text{(Eq 7)}$$

The relationship between the responses, compression strength and mould hardness in the cement-bonded moulding sand system is shown in Fig. 7. Compression strength was expressed as a 3rd order non-linear function of the mould hardness by determining the best-fit curve corresponding to one thousand points representing the compression strength-mould hardness relationships in 2D. The coefficient of correlation was found to be equal to 0.986, which indicates a strong non-linear relationship between the compression strength and mould hardness. It is observed from this figure that the compression strength of the sand mould will increase, as its hardness increases.

4. Concluding Remarks

Early strength development of the cement-bonded mould was possible by the use of an accelerator. Both linear as well as non-linear regression analyses were conducted to represent the mould properties like compression strength and hardness as the functions of the process parameters, namely % cement, % accelerator, cement-to-water ratio, and testing time. The effects of different process parameters on the responses were studied with the help of main effect and surface plots. Compression strength of the mould was influenced much by the parameters, testing time and % cement. It was found to increase with the increase of the above process parameters and decrease of cement-to-water ratio. Compression strength had a linear relationship with % cement and % accelerator, and non-linear relationship with the other parameters, namely cement-to-water ratio and testing time. Mould hardness was seen to increase with the increase of % cement, % accelerator, and testing time, whereas it had a negative relationship with the parameter called cement-to-water ratio. Moreover, it was seen to have linear relationship with the parameter, testing time, and non-linear relationship with the remaining parameters. It was noted that compression strength of the mould has 3rd order polynomial relationship with its hardness.

The statistical adequacy of each model was tested through ANOVA test. It was found that all the developed models were

Table 5 **Summary of the results of test cases for the response-mould hardness**

Test No.	Measured hardness	Mould hardness using full-factorial design				Mould hardness using central composite design			
		Full-Fact. design	Deviation	% Deviation	Abs % deviation	CCD	Deviation	% Deviation	Abs % deviation
1	70.75	68.31	2.44	3.44	3.44	71.12	−0.37	−0.52	0.52
2	80.75	74.76	5.99	7.42	7.42	78.15	2.60	3.22	3.22
3	83.00	78.97	4.03	4.86	4.86	82.59	0.41	0.49	0.49
4	89.75	88.63	1.12	1.25	1.25	90.20	−0.45	−0.50	0.50
5	87.75	83.42	4.33	4.94	4.94	87.74	0.01	0.01	0.01
6	83.50	80.79	2.71	3.24	3.24	83.71	−0.21	−0.25	0.25
7	84.00	81.88	2.12	2.53	2.53	82.67	1.33	1.59	1.59
8	77.00	73.26	3.74	4.86	4.86	76.26	0.74	0.96	0.96
9	80.50	75.45	5.05	6.27	6.27	80.28	0.22	0.27	0.27
10	84.25	80.45	3.80	4.52	4.52	85.31	−1.06	−1.25	1.25
11	87.75	87.13	0.62	0.71	0.71	88.50	−0.75	−0.86	0.86
12	88.00	84.98	3.02	3.43	3.43	85.70	2.30	2.62	2.62
13	84.75	80.79	3.96	4.67	4.67	83.71	1.04	1.23	1.23
14	82.25	79.97	2.28	2.77	2.77	83.43	−1.18	−1.44	1.44
15	81.50	76.63	4.87	5.98	5.98	80.19	1.31	1.61	1.61
16	72.50	71.64	0.86	1.19	1.19	74.20	−1.70	−2.35	2.35
17	77.00	77.38	−0.38	−0.49	0.49	79.00	−2.00	−2.60	2.60
18	78.50	74.83	3.67	4.67	4.67	79.64	−1.14	−1.46	1.46
19	89.25	86.40	2.85	3.20	3.20	89.10	0.15	0.17	0.17
20	82.75	81.56	1.19	1.43	1.43	83.04	−0.29	−0.35	0.35
	Abs. % Error				71.87				23.74
	Avg. of % Error				3.59				1.19

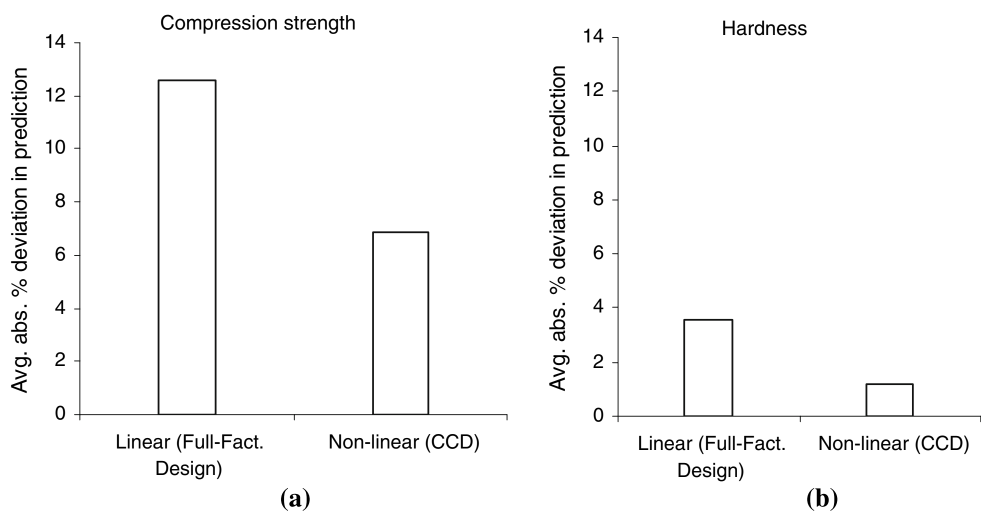

Fig. 6 Comparison of the models in terms of average of absolute % deviation in prediction of the response values

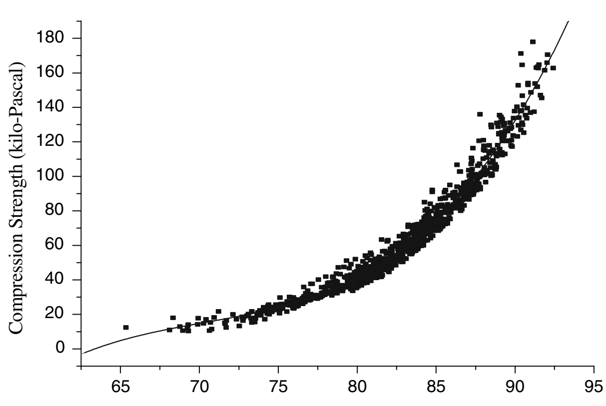

Fig. 7 Third-order regression model to establish a relationship between compression strength and mould hardness

statically adequate and could be used for making the predictions. Further, the developed models were tested for their practical significance with the help of 20 randomly generated test cases. The non-linear regression model based on 3-level CCD was found to be better than the linear model based on 2-level full-factorial design for the responses, compression strength and mould hardness. This supremacy of non-linear model over the linear one might be due to the non-linear relationships of the inputs with the responses.

Acknowledgments

The first author gratefully acknowledges the help of Quality Improvement Program, Ministry of Human Resource Development, Govt. of India and Gogte Institute of Technology, Belgaum, India.

Appendix A: Design Matrix for Full-Factorial Design

	Input parameters				Responses	
Sl. No.	*A*	*B*	*C*	*D*	Compression strength, kPa	Hardness
1	−1	−1	−1	−1	-	-
2	+1	−1	−1	−1	-	-
3	−1	+1	−1	−1	-	-
4	+1	+1	−1	−1	-	-
5	−1	−1	+1	−1	-	-
6	+1	−1	+1	−1	-	-
7	−1	+1	+1	−1	-	-
8	+1	+1	+1	−1	-	-
9	−1	−1	−1	+1	-	-
10	+1	−1	−1	+1	-	-
11	−1	+1	−1	+1	-	-
12	+1	+1	−1	+1	-	-
13	−1	−1	+1	+1	-	-
14	+1	−1	+1	+1	-	-
15	−1	+1	+1	+1	-	-
16	+1	+1	+1	+1	-	-

Appendix B: Design Matrix for Central Composite Design

	Input parameters				Responses	
Sl. No.	*A*	*B*	*C*	*D*	Compression strength, kPa	Hardness
1	−1	−1	−1	−1	-	-
2	+1	−1	−1	−1	-	-
3	−1	+1	−1	−1	-	-
4	+1	+1	−1	−1	-	-
5	−1	−1	+1	−1	-	-
6	+1	−1	+1	−1	-	-
7	−1	+1	+1	−1	-	-
8	+1	+1	+1	−1	-	-
9	−1	−1	−1	+1	-	-
10	+1	−1	−1	+1	-	-
11	−1	+1	−1	+1	-	-
12	+1	+1	−1	+1	-	-
13	−1	−1	+1	+1	-	-
14	+1	−1	+1	+1	-	-
15	−1	+1	+1	+1	-	-
16	+1	+1	+1	+1	-	-
17	0	0	0	0	-	-
18	−1	0	0	0	-	-
19	+1	0	0	0	-	-
20	0	−1	0	0	-	-
21	0	+1	0	0	-	-
22	0	0	0	0	-	-
23	0	0	−1	0	-	-
24	0	0	+1	0	-	-
25	0	0	−1	0	-	-
26	0	0	0	+1	-	-
27	0	0	0	0	-	-

Appendix C: Input-Output Data of the Test Cases

Test No.	*A* % cement	*B* accelerator	*C* cement-to-water ratio	*D* testing time, h	Average CS, kPa	Average mould hardness
1	10	2.5	2.5	2	16.20	70.75
2	8	3.5	2.0	4	33.61	80.75
3	9	2.0	2.0	8	73.77	83.00
4	11	3.5	1.5	7	130.14	89.75
5	11	2.5	2.0	7	92.73	87.75
6	12	2.5	2.0	4	65.33	83.50
7	12	4.0	2.5	5	73.60	84.00
8	9	2.5	2.5	7	35.51	77.00
9	9	3.0	2.0	4	41.02	80.50
10	9	2.5	2.0	8	82.05	84.25
11	11	3.5	1.5	6	111.01	87.75
12	10	2.0	1.5	7	92.56	88.00
13	12	2.5	2.0	4	73.08	84.75
14	12	3.0	2.0	3	63.43	82.25
15	8	2.5	2.0	7	52.92	81.50
16	10	2.5	2.5	4	20.17	72.50
17	10	2.0	1.5	2	29.30	77.00
18	10	2.5	2.0	3	35.51	78.50
19	10	3.0	1.5	7	104.28	89.25
20	9	2.5	1.5	5	52.57	82.75

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